RG Salinity Variance + Mixing

- what is salinity variance?
- Why do we want to calculate it?

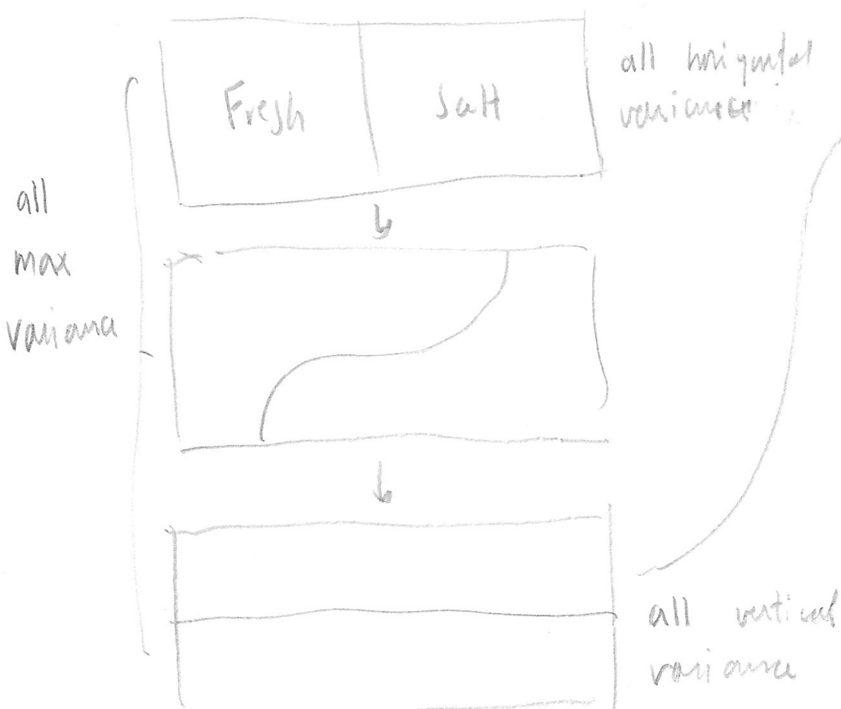
$$SVAR = \frac{1}{V}\iiint (s - \langle s \rangle)^2 \, dV$$

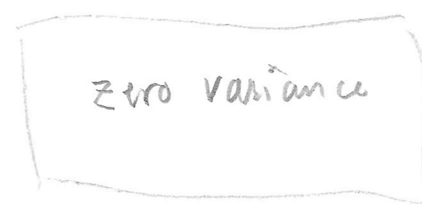
$$= \langle s'^2 \rangle$$

$\langle \, \rangle$ = volume average

$s' = s - \langle s \rangle$

Lock exchange:





Typically in estuaries it is dominated by horizontal variance.

Deriving the variance equation

$$2S'\left[S_t + \underline{u}\cdot\nabla S = (K S_z)_z \right] \qquad S' = \langle S \rangle + S'$$

Note all the terms multiplied by $\langle S \rangle$ will add to zero

$$\Rightarrow \quad S'^2_t + \underline{u}\cdot\nabla S'^2 = 2S'(K S'_z)_z$$

$$= \left[K(S'^2)_z\right]_z - 2K(S'_z)^2$$

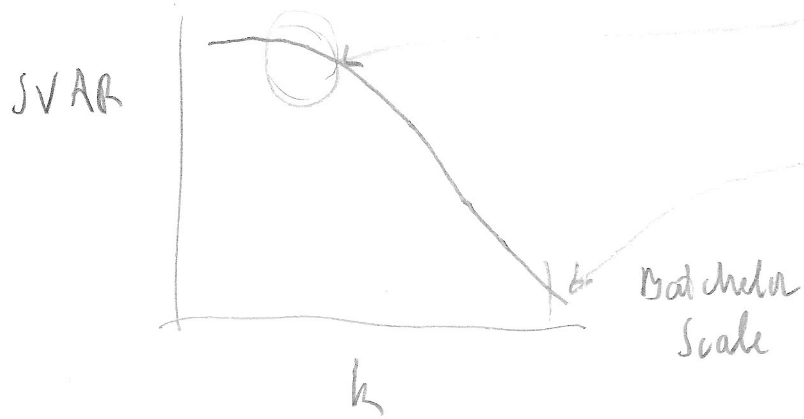
$$\Rightarrow \quad \underbrace{S'^2_t + \underline{u}\cdot\nabla S'^2 = (K S'^2_z)_z}_{\text{looks a lot like the original S equation, but for } S'^2} \underbrace{- 2K(S'_z)^2}_{\text{Mixing}}$$

looks a lot like the original S equation, but for S'^2

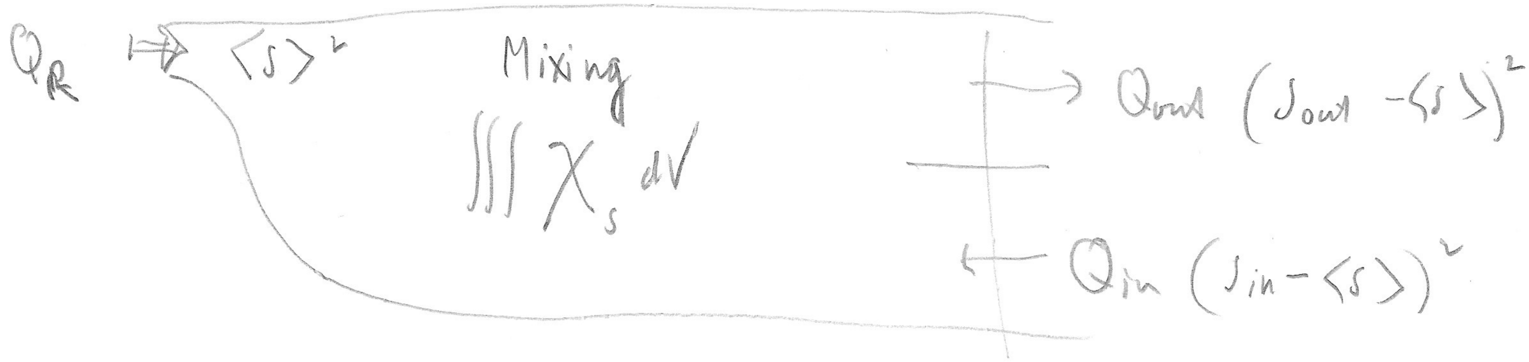
Mixing

A New term.

Really production of variance at scales where turbulence & molecular processes can destroy it.



$$\text{Translation} \quad \underbrace{2K\left(\frac{\partial S}{\partial z}\right)^2}_{\text{like production}} = \underbrace{\chi_s = \overbrace{2K(\nabla S)^2}^{\text{turbulence}}}_{\text{like dissipation}}$$



variance $= (s - \langle s \rangle)^2$

$$\overset{+}{Q_R} \langle s \rangle^2 + \overset{+}{Q_{in}} (S_{in} - \langle s \rangle)^2 + \overset{-}{Q_{out}} (S_{out} - \langle s \rangle)^2 = \text{Mixing}$$

(assumes steady state - not true for different reasons

Baltic: interannual

Ches. + PS: annual

Hudson: spring neap

$Q_R + Q_{in} + Q_{out} = 0$

$Q_{in} S_{in} + Q_{out} S_{out} = 0$

use these to remove terms like $\langle s \rangle^2 + 2 s' \langle s \rangle$

using them

$$\boxed{Q_{in} S_{in}^2 + Q_{out} S_{out}^2 = \text{Mixing}}$$

~~because Q~~

Using Knudsen

$$\boxed{Q_R \, S_{in} \, S_{out} = \text{Mixing}}$$

and

$$Q_{in} S_{in} \Delta S = \text{Mixing}$$

$$- Q_{out} S_{out} \Delta S = \text{Mixing}$$

Exploring this: imaging Q_R fixed, $S_{in} = S_{ocn}$

for very small mixing $S_{out} \to 0$

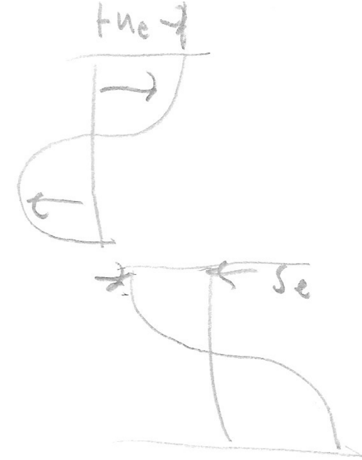
Also max mixing is $Q_R S_{ocn}^2$

☆ Exercise

$$u_E \sim \frac{1}{48} \beta g \frac{\partial s}{\partial x} \frac{h^3}{K}$$

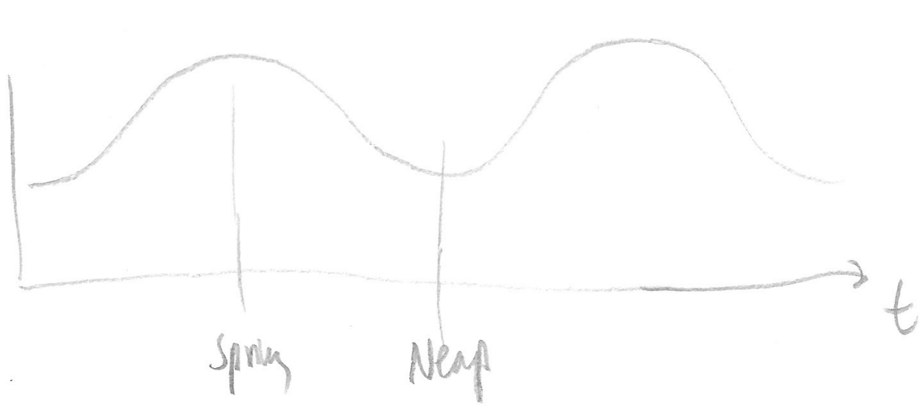
$$S_E \sim u_E \frac{\partial s}{\partial x} \frac{h^2}{K}$$

$$\text{Mixing} = \iiint K \left(\frac{\partial s}{\partial z}\right)^2 dV$$



Hudson

$\frac{\partial s}{\partial x}$ fixed



① Q is there more mixing during spring or neap?

② What is power law relation between Mixing and $\frac{\partial s}{\partial x}$?

Answer

$$M \sim K\left(\frac{\partial s}{\partial t}\right)^2 \qquad \frac{\partial s}{\partial t} \sim \frac{s_\epsilon}{h} \sim \left[u_\epsilon \frac{\partial s}{\partial x} \frac{h}{K}\right]$$

$$\Rightarrow M \sim K\left(u_\epsilon \frac{\partial s}{\partial x} \frac{h}{K}\right)^2$$

$$\frac{1}{H^3}$$

$$a \quad M = K\left(\cancel{\beta} g \frac{\partial s}{\partial x} \frac{h^3}{K} \frac{\partial s}{\partial x} \frac{h}{K}\right)^2$$

$$\frac{M}{\beta g^2} = \cancel{K} \frac{\partial s}{\partial x}^4 \frac{h^6}{K^2} \frac{h^2}{\cancel{K^2}} \sim \frac{h^8}{K^3}\left(\frac{\partial s}{\partial x}\right)^4$$

$\Rightarrow$ 1 More mixing during Neap (K lower $\Rightarrow$ Mix higher)

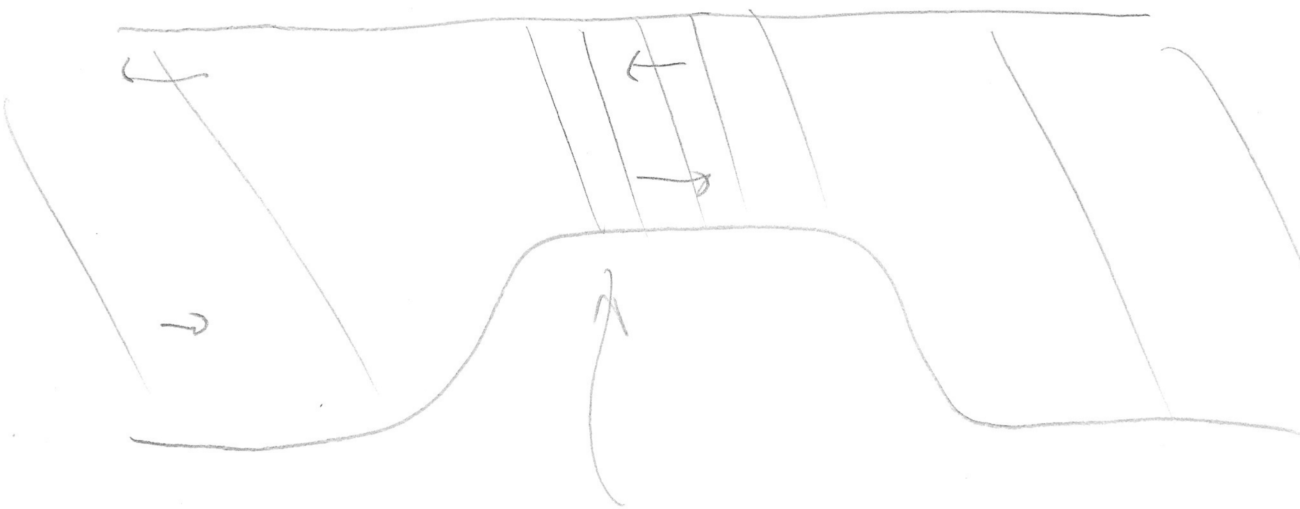
2 Mix $\propto \left(\frac{\partial s}{\partial x}\right)^4$!

(Interesting to consider how this varies with H)

$$\frac{\partial s}{\partial x} \sim \frac{1}{H^3} \Rightarrow \text{Mix} \sim \frac{h^8}{h^{12}} \sim \frac{1}{H^4} \ ?$$

Thinking about this ...

thinking about $Mix \sim \left(\frac{\partial s}{\partial x}\right)^4$ and $\frac{\partial s}{\partial x} \sim \frac{1}{H^3}$ ⑥



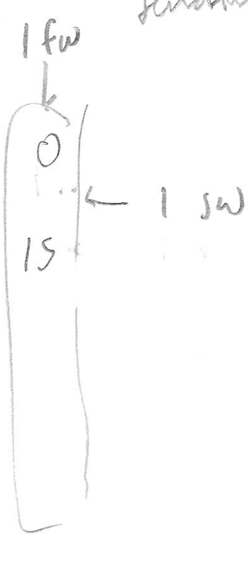
$Mix \sim \frac{1}{H^4}$ $\Rightarrow$ <u>much</u> more mixing

over shallow places

- Where does variance come from?

The hydrologic cycle driven by the sun.

The sun is a little chunk of the big bang, sending low-entropy photons to the Earth.



0 → 15

15 — 30

Total Salinity Variance $= \int\!\!\int\!\!\int\!\!\int (S'^2)\, dV = SVAR$

(vertical average of S')

Horizontal Variance $= \iiint (\overline{S'})^2\, dV = SVAR_H$

(start w/ $\bar{S}$)

Vertical Variance $SVAR_V = SVAR - SVAR_H$

Total variance tendency

$$\overline{S'^2_t} + \overline{\underline{u} \cdot \nabla S'^2} = -2K\overline{(S'_z)^2}$$

Horizontal variance tendency

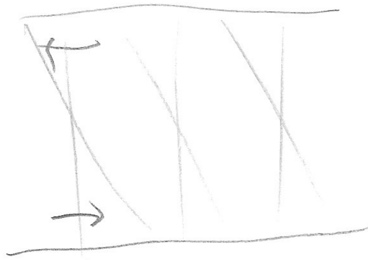
$$\overline{S'}^2_t + \underline{u} \cdot \nabla \overline{S'}^2 = 2\,\overline{\underline{u}'S'} \cdot \overline{\nabla_H S}$$

Transfer of variance vert $\leftrightarrow$ horiz

Subtracting Horizontal from total gives Vertical variance tendency

$$\overline{S'^2_v} + \overline{\underline{u} \cdot \nabla_H S'^2_v} = -2\,\overline{\underline{u}'S'} \cdot \overline{\nabla_H S} - 2K\overline{(S'_z)^2}$$

Note: mixing can't touch horizontal variance straining



$$\frac{\partial}{\partial z}\left(\frac{\partial s}{\partial t} = -u\frac{\partial s}{\partial x}\right)$$

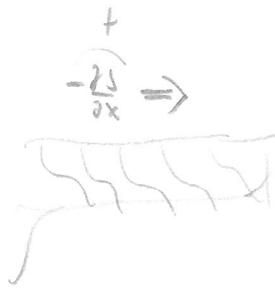
$$\Rightarrow \frac{\partial}{\partial t}\frac{\partial s}{\partial z} = \underbrace{-\frac{\partial u}{\partial z}\frac{\partial s}{\partial x}}_{\text{STRAINING}} - \underbrace{u\frac{\partial}{\partial x}\frac{\partial s}{\partial z}}_{\text{(advection of } \partial s/\partial z}$$

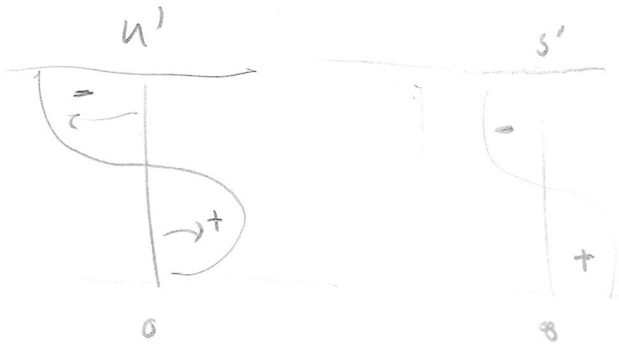
units $[\text{psu}\ L^{-1}\ T^{-1}]$

vs. $(\text{psu})^2\ T^{-1}$ in the variance equation

Note: straining term in variance equation

is $-\overline{u's'}\frac{\partial s}{\partial x}$ ~ $\overset{+}{\text{exchange flow salt flux}} \times \overset{+}{\left(-\frac{\partial s}{\partial x}\right)}$





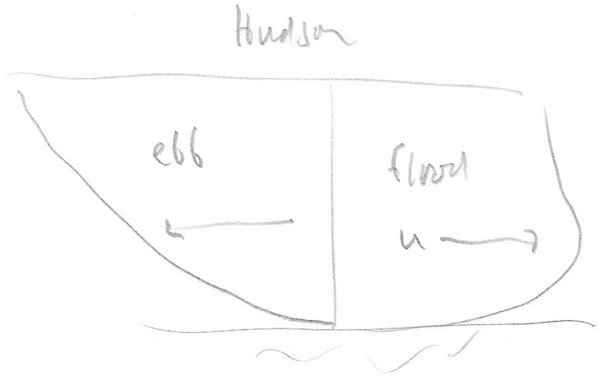
× estuarine salt flux in the down-gradient direction

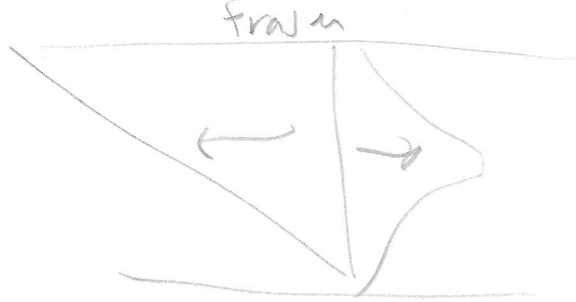
and since $\overline{u's'} \sim (\bar{s}_x)^3$

the straining goes like $(\bar{s}_x)^4$

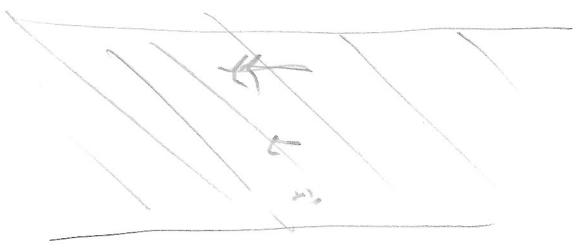
straining increases vertical variance at the expense of horizontal variance.

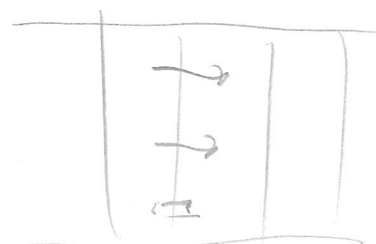
Tidal Straining





Bottom half of the water column: shear changes sign
flood ⟷ ebb.





★ [Reverse (negative) straining doesn't really happen very much.]